Static and non-static quantum effects in two-dimensional dilaton gravity

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Abstract

We study backreaction effects in two-dimensional dilaton gravity. The backreaction comes from an R^2 term which is a part of the one-loop effective action arising from massive scalar field quantization in a certain approximation. The peculiarity of this term is that it does not contribute to the Hawking radiation of the classical black hole solution of the field equations. In the static case we examine the horizon and the physical singularity of the new black hole solutions. Studying the possibility of time dependence we see the generation of a new singularity. The particular solution found still has the structure of a black hole, indicating that non-thermal effects cannot lead, at least in this approximation, to black hole evaporation.

1   Introduction

Two-dimensional dilaton gravity is a useful laboratory for addressing fundamental questions of quantum gravity. It exhibits many of the interesting non-trivial features of four dimensional gravity, describing the s-wave sector of four dimensional Einstein gravity, and especially black hole solutions [1] permiting a study of the interplay between gravity and quantum mechanics. This fact combined with the stringy origin of the action justifies the persisting activity in the field.

In a previous work [2] the Hawking radiation due to the presence of quantum scalar massive particles has been calculated. One of the results of this work was that purely geometrical terms (not including the dilaton field) in the one-loop effective action do not contribute to the thermal radiation. This observation raises the question of the evolution of a Schwarzschild black hole in the presence of such terms. The first of these terms in the one-loop effective action, in a large mass expansion, is just an R^2 term. Thus two-dimensional higher derivative gravity is interesting for one more reason. It can in principle give information about non-thermal effects of quanrum origin, in black hole pfysics. Backreaction effects including this term were considered in [3], where static black hole geometries were found as solutions of the field equations of the two dimensional dilaton gravity with R^2 term. The existence of these solutions viewed as backreaction effects is compatible with the fact that the extra terms do not contribute to the Hawking radiation but in no way can say something about the evolution of the original black hole like the case, for example, of the Russo, Susskind, Thorlacius model [4].

In this work we try to incorporate time dependence in the backreaction effects due to the presence of the R^2 term. In section 2 we formulate the problem and give the field equations both in covariant form and in the conformal gauge using light cone coordinates, which are more convenient to incorporate time dependence. In section 3 we point out the basic features of the static solutions found in [3]. The analysis here is presented in the abovementioned coordinate system giving emphasis in the study of the region behind the event horizon and in the appearence of the singularity. This discussion completes the results of [3]. In section 4 we consider the non-static case. We find time-dependent solutions expanding in powers of the coefficient (κ) of the R^2 term . In κ^2 order we find solutions with black hole structure differing from the static case mainly in the appearence of an extra physical singularity. Up to this order the solution found has striking similarity to the one found in [5]. In this reference the corresponding solution was an exact solution of the field equations in the presence of non-trivial (classical) tachyon configuration. We conclude with a discussion of our results in section 5.

2   Setting the problem

The classical action of a scalar field coupled to the two-dimensional gravity (and to the dilaton field) is

$$S_{cl} = \frac{1}{2\pi} \int \sqrt{-g} e^{-2\phi} \{ [R + 4(\nabla\phi)^2 + 4\lambda^2] - [(\nabla T)^2 - m_0 T^2] \} \tag{1}$$

while the rescaling $T = e^{-\phi}\tilde{T}$ giving to the scalar field canonical kinetic terms yields:

$$S_{cl} = \frac{1}{2\pi} \int \sqrt{-g} e^{-2\phi}\{[R + 4(\nabla\phi)^2 + 4\lambda^2] - [(\nabla\tilde{T})^2 + ((\nabla\phi)^2 - \Box\phi - m_0^2)\tilde{T}^2]\}. \tag{2}$$

Note that the term $((\nabla\phi)^2 - \Box\phi - \lambda^2)\tilde{T}^2$ gives the quadratic coupling of the scalar field to the dilaton, being zero in the linear dilaton vacuum, while $m^2 = \lambda^2 - m_0^2$ is the "mass" of the scalar field.

Quantizing the scalar field one can take the one-loop effective action, which contains both local and non-local terms [6], [2]. Expanding the purely geometric part of the non-local terms (which are responsible for the Hawking radiation) in inverse powers of the mass m^2 and keeping the first term [3] we get

$$S = S_{cl} - \kappa \int \sqrt{-g} R^2 \tag{3}$$

where $\kappa = 1/(240m^2)$. Since we are not interested in this work in non-trivial scalar field configurations we consider the metric and dilaton equations which read:

$$- 2e^{-2\phi}[g_{\mu\nu}((\nabla\phi)^2 - \Box\phi - 1) + \nabla_\mu\nabla_\nu\phi] - \kappa[2\nabla_\mu\nabla_\nu - \frac{g_{\mu\nu}}{2}R^2 - 2g_{\mu\nu}\Box R] = 0, \tag{4}$$

$$e^{-2\phi}\{\frac{R}{4} - [(\nabla\phi)^2 - \Box\phi - 1]\} = 0 \tag{5}$$

In the following we'll work in the conformal gauge and light-cone coordinates (x^+, x^-) where the line element reads

$$ds^2 = -e^{2\rho}dx^+dx^- \tag{6}$$

and the components of the Ricci tensor take the form $R_{++} = -2\partial_+^2\rho$, $R_{--} = -2\partial_-^2\rho$ and $R = 8e^{-2\rho}\partial_+\partial_-\rho$ while for a scalar field f we have $\Box f = -4e^{-2\rho}\partial_+\partial_-f$.

In this gauge the dilaton equation becomes

$$e^{-2\phi}\{1 + 2e^{-2\rho}\partial_+\partial_-\rho + 4e^{-2\rho}[\partial_+\phi\partial_-\phi - \partial_+\partial_-\phi]\} = 0 \tag{7}$$

while from the equations for the metric components we get the trace equation

$$- e^{2(\rho-\phi)} + e^{-2\phi}(-4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\phi) + \kappa(2\partial_+\partial_-R - \frac{1}{4}e^{2\rho}R^2) = 0 \tag{8}$$

and the two constraints

$$e^{-2\phi}(2\partial_-^2\phi - 4\partial_-\phi\partial_-\rho) + \kappa(4\partial_-\rho\partial_-R - 2\partial_-^2R) = 0 \tag{9}$$
$$e^{-2\phi}(2\partial_+^2\phi - 4\partial_+\phi\partial_+\rho) + \kappa(4\partial_+\rho\partial_+R - 2\partial_+^2R) = 0 \tag{10}$$

Note that combining the trace (8) and the dilaton (7) equations we get the relation

$$2e^{-2\phi}\partial_+\partial_-(\rho - \phi) = \kappa[16e^{-2\rho}(\partial_+\partial_-\rho)^2 - 16\partial_+\partial_-(e^{-2\rho}\partial_+\partial_-\rho)]. \tag{11}$$

We see that when $\kappa = 0$ we recover the $\rho - \phi$ symmetry [1],[4]. The system admits in this case the solution

$$e^{-2\phi} = e^{-2\rho} = M - x^+x^- \qquad (12)$$

for the conformal factor and the dilaton field while for the Ricci scalar we take:

$$R_{\kappa=0} = 4Me^{2\rho}. \qquad (13)$$

The solution in (12), unique in the $\kappa = 0$ case, describes either a static black hole solution ($M \neq 0$) or flat space ($M = 0$) [1]. The presence of the backreaction terms ($\kappa \neq 0$), spoils the $\rho - \phi$ symmetry leading to new static solutions and permitting time dependence.

3 Static Solutions

Static black hole solutions emerging from the Lagrangian in (3) have already been found in [3] (and in somehow different context in [7]) where the Schwarzschild coordinate system was adopted. In this section we will reconsider the basic features of these solutions working in the conformal gauge mainly for use in the time dependent case.

For static solutions we consider the fields as functions of the combination $-x^+x^-$ only. This is clear from the fact that the coordinates $x^\pm$ are related to the physical spacetime conformal coordinates through the relations $x^+ = e^{\sigma^+}$, $x^- = -e^{\sigma^-}$, where $\sigma^\pm = \frac{1}{2}(t \pm x)$, or conversely $x = ln(-x^+x^-)$, $t = -ln(-\frac{x^-}{x^+})$.

The system of the equations is a fourth order one, due to the presence of the R^2 term in the action. In order to reduce the order we assume the field ρ is monotonic function of the variable x^+x^-. This is always possible since the number of degrees of freedom (ρ and ϕ in our case) is less than the number of equations so one of the degrees of freedom can have this property. Note that the dilaton field cannot be a monotonic function as we have seen from the analysis in [3]. The above monotonicity property is also true in four-dimensional black hole solutions [8]. The non-monotonicity of the dilaton is explained by the fact that the R^2 forces are repulsive. Furthermore we introduce the "auxiliary" field $A(\rho)$ through the relation:

$$R(x^+, x^-) = 8e^{-2\rho}\partial_+\partial_-\rho = 8e^{-2\rho}A(\rho). \qquad (14)$$

and we also consider the dilaton field being function of ρ, $\phi(\rho)$. Writting the system of equations in terms of $A(\rho)$ and $\phi(\rho)$ and eliminating the derivatives of ρ we are left with an ordinary non-linear system. For this system we seek solutions analytic in the parameter κ written in the form of κ-series as

$$\begin{align}
A(\rho) &= A_1(\rho) + \kappa A_2(\rho) + ... \\
\phi(\rho) &= \rho + \kappa H_1(\rho) + \kappa^2 H_2(\rho) + ...
\end{align} \qquad (15)$$

where $H(\rho) = \kappa H_1(\rho) + \kappa^2 H_2(\rho) + \ldots$ denotes the deviation from the $\rho - \phi$ symmetry in the presence of the R^2 terms. For convenience in the integration of the equations we use one more auxiliary field:

$$h(\rho) = e^{-2\rho} - F(\rho; \kappa) = M - x^+ x^-. \tag{16}$$

The solution of the resultant system up to κ^4 -order reads:

$$A(\rho) = M_{10}e^{4\rho}[\frac{1}{2} + 16\kappa e^{2\rho} + 16\kappa^2(96 - M_{10})e^{4\rho} + \frac{64(41472 - 733M_{10})}{9}\kappa^3 e^{6\rho}]$$

$$\phi(\rho) = \rho + \kappa M_{10}e^{4\rho}[1 + \frac{512}{9}\kappa e^{2\rho} + \kappa^2\frac{41472 - 437M_{10}}{6}e^{4\rho}] \tag{17}$$

while the dependence of the fields on the spacetime coordinates is given through the field $h(\rho)$ up to κ^2-order by

$$h(\rho) = e^{-2\rho} - 8\kappa^2 M_{10}e^{2\rho} + O(\kappa^3) = M - x^+ x^- \tag{18}$$

The expression of the solutions in κ-expansion will be proven helpful in the time-dependent case but since it appears arbitrary [1] we will justify it before proceeding in the discussion of the solutions. Since the solutions we seek for have to meet the asymptotic flatness condition, the fields $A(\rho)$, $H(\rho)$ have vanishing limit as $\rho \to -\infty$. So we can solve the nonlinear system of equations by iteration following the method adopted in [3]. The linearized system reads:

$$H''' = 2e^{-2\rho}(A' - 4A) + 8\kappa(-A'' + 4A' - 4A) + H''$$

$$A'' = 4A' - 4A + \frac{1}{8\kappa}H'' \tag{19}$$

which after the elimination of the field H yields an equation for A namely

$$(4A - A') + 4\kappa e^{2\rho}(4A' - 4A'' + A''') = 0. \tag{20}$$

The solution of this equation is:

$$A(z) = \frac{c_1}{z}I_1(z) + \frac{c_2}{z}K_1(z) + c_3 Im[\frac{1}{z}S_{-2,1}(iz)] \tag{21}$$

where $z = \frac{e^{-\rho}}{2\sqrt{\kappa}}$, I_1, K_1 are the modified Bessel functions, $S_{-2,1}$ is the Lommel function and the c_i's are integration constants.

Now if we write the system as a quasi-linear first order system

$$\vec{X}'(\rho) = \mathbf{B}(\rho)\vec{X}(\rho) + \vec{\mathbf{F}}(\vec{X}), \tag{22}$$

where $\mathbf{B}$ is a 5×5 matrix giving the linear part of the system,

$$\vec{\mathbf{F}}^{\mathrm{T}} = (0, 0, \mathbf{F}^H, 0, \mathbf{F}^A) \tag{23}$$

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[1] In fact analitycity in κ comes from the general theory for non-linear ordinary differential systems.

denotes the non-linear terms, and

$$\vec{X}(\rho)^{\mathrm{T}} = (H(\rho), H^{'}(\rho), H^{''}(\rho), A(\rho), A^{'}(\rho)). \tag{24}$$

The solution in (21) can be used to produce a fundamental solution $\mathbf{Y}(\rho)$ of the linear part of (22). Then the full (asymptotically stable) solution is given iteratively in the form:

$$\vec{X}_n(\rho) = \vec{X}_{(n-1)}(\rho) + \int^{\rho} \mathbf{Y}(\rho)\mathbf{Y}^{-1}(t)\vec{\mathbf{F}}[\vec{X}_{(n-1)}(t)]dt \tag{25}$$

We don't present here the corresponding expressions because they are too lengthy. Working out the first iteration we see that the κ-series solution (17) coincides with the asymptotic expansion of the exact solution taken by the iteration scheme confirming, at least asymtotically, the analyticity in the parameter κ.

Note that the position of the apparent horizon (coinciding with the event horizon in the static case) is given by

$$\partial_+ \rho = -x^- J(\rho) = 0, \tag{26}$$

where $J(\rho) = \frac{1}{\partial_\rho h(\rho)}$. In the case of the C.G.H.S. black hole [1] we have $J_c(\rho) = -\frac{1}{2}e^{2\rho}$ leading to the unique apparent horizon, coinciding with the event horizon, at $x_- = 0$. In our case the function $J(\rho)$ is more complicated leaving room for other solutions of (26). Substituting the derivatives of ρ from the constaints in (10) we get the equation for the field $J(\rho)$ which keeping terms up to second order reads

$$J^{'}(\rho) = 2J(\rho) - 2J(\rho)H^{''}(\rho). \tag{27}$$

From the second of equations in (19) and taking into account that $R^{(2)}(\rho) = 8e^{-2\rho}A(\rho)$ the equation (19) can be written as

$$J^{'}(\rho) = 2J(\rho) - 2\kappa J(\rho)e^{2\rho}R^{(2)''}(\rho). \tag{28}$$

The solution of the linear part is taken to be $J_{lin}(\rho) = J_c(\rho) = -\frac{1}{2}e^{2\rho}$. Since the field J does not appear in the system (19) it is easy to enlarge the system including also the equation (28) and to work out the first iteration. Thus we can take an expression for the field $J(\rho)$. We see now that $J(\rho)$ has a zero at some $x^+x^- = constant < 0$. The new apparent horizon appears when $J(\rho_H) = 0$. So we see that the backreaction effects shift the horizon in the physical spacetime region of the C.G.H.S. case namely on an hyperbola $x^+x^- = constant < 0$.

In order to investigate the existence of the singularity we will make some comments for the function $h(\rho)$ (in fact the implicit dependence of ρ on the space coordinate). Using the ansatz in (15) in the form $\phi(\rho) = \rho + H(\rho)$ the dilaton equation is written as:

$$1 + \partial_+\partial_- e^{-2\rho} + e^H \partial_+\partial_- \{\int^{\rho} e^{-2y-H(y)}[-4H'(y)]dy\} = 0. \tag{29}$$

Multiplying by appropriate factor the above equation can be formally integrated to give:

$$h(\rho) = e^{-2\rho} + \int^{\rho} e^{-2y-F(y)}[-4H'(y)]dy \tag{30}$$

where $F(\rho)$ satisfies the equation

$$e^{-F-H}[-F'(H' + 2H'^2) + (2H'^3 + H'')] + (e^{-H})'' = 0. \tag{31}$$

This equation can be solved exactly to give the function $F(\rho)$ in terms of $H(\rho)$. Instead of giving the lengthy expression for F we investigate its behaviour near the singularity $(\rho \to \infty)$. The relevant terms in (31) give the (asymptotic) equation

$$e^{-F}F'(-2H'^2) + e^{-F}(2H'^3) = 0 \tag{32}$$

with solution

$$F_{sing}(\rho) \sim H_{sing}(\rho). \tag{33}$$

So the function $h(\rho)$ behaves near the singularity as:

$$h_{sing}(\rho) = e^{-2\rho} + 4e^{-2\rho-H} - 2\int^{\rho} e^{-2y-H(y)}dy. \tag{34}$$

Since the function $e^{-H(\rho)}$ vanishes very rapidly near the singularity ($H(\rho) \to +\infty$ as $\rho \to +\infty$), we see that $h(\rho)$ remains a positive function going to zero as $\rho \to +\infty$. From the definition of $h(\rho)$ in (16) we see that this happens when $x^+x^- = M$.
As far as the asymptotically flat region is concerned ($\rho \to -\infty$) the equation (31) is approximated by:

$$e^{-F}(H'' - F'H') - H'' = 0 \tag{35}$$

admitting the general solution

$$F_{as}(\rho) = -Log\left[\frac{H'(\rho) - c}{H'(\rho)}\right] \tag{36}$$

which with the choice $c = 0$ gives

$$F_{as}(\rho) = 0. \tag{37}$$

Then as is easily recognized from the dilaton equation, keeping only linear terms in the field $H(\rho)$,

$$h_{as}(\rho) = e^{-2\rho} + 2\kappa e^{-2\rho}\int^{\rho} e^{2y}R''(y)dy - 2\kappa R'(\rho) \tag{38}$$

where we have used the fact that asymptotically

$$H''(\rho) = \kappa e^{2\rho}R''(\rho). \tag{39}$$

Substituting from (21) the expression $R \sim zIm[S_{-2,1}(iz)]$ we find that

$$h(z) = 4\kappa\left[z^2 - \frac{z^2}{2}\int^z \frac{Re[S_{-1,0}(iu)]}{u}du\right]. \tag{40}$$

The above analysis convinces about the monotonicity of ρ as a function of x^+x^- which was anticipated in the begining of this section. For the singularity we have to make two

comments. The first is that in this analysis comes out naturally that the geometry of the spacetime has a physical singularity. This could not be seen from the numerical solutions in the Schwarzschild gauge as e.g. in [9], [3]. The second remark is that unlike the C.G.H.S. case the constant M appearing in (16) is not the mass of the black hole. Thus it can take negative values also. This means that there exist R^2 black hole solutions which have both their horizon and their singularity lying in different hyperbolae in the physical spacetime region of the C.G.H.S. geometry.

4 Time dependence

In order to incorporate the time dependence we keep ρ as one of the variables but we allow explicit dependence on x^+ also. In particular the implicit dependence of ρ on the variable x^+x^- of the static case (16) is replaced now by the ansatz:

$$h(x^+, \rho) = G(x^+) - x^- F(x^+) \tag{41}$$

where $F(x^+)$ must be a monotonic function and the left hand side has the form:

$$h(x^+, \rho) = e^{-2\rho} + \kappa h_1(x^+, \rho). \tag{42}$$

When $\kappa = 0$ we get the well known solution with $G(x^+)$ constant and $F(x^+) = x^+$. Having in mind an evolution scheme we make the following ansatz for the fields ϕ and $\partial_+\partial_-\rho$:

$$\phi = \rho + f_{st}(\rho; \kappa) + \kappa H(x^+, \rho) \tag{43}$$

$$\partial_+\partial_-\rho = A_{st}(\rho; \kappa) + \kappa A(x^+, \rho) \tag{44}$$

where $f_{st}(\rho; \kappa)$ and $A_{st}(\rho; \kappa)$ are the solutions found in the static case.
Now the system of equations (7, 8, 10) becomes:

$$1 + B_1^{(dil)}(x^+, \rho)(\partial_-\rho) \ +$$
$$B_2^{(dil)}(x^+, \rho)(\partial_-\rho)(\partial_+\rho) + B_3^{(dil)}(x^+, \rho)(\partial_-\partial_+\rho) = 0,$$
$$B_0^{+-}(x^+, \rho) + B_1^{+-}(x^+, \rho)(\partial_-\rho) \ +$$
$$B_2^{+-}(x^+, \rho)(\partial_-\rho)(\partial_+\rho) + B_3^{+-}(x^+, \rho)(\partial_-\partial_+\rho) = 0,$$
$$B_2^{--}(x^+, \rho)(\partial_-\rho)^2 + B_3^{--}(x^+, \rho)(\partial_-^2\rho) = 0,$$
$$B_0^{++}(x^+, \rho) + B_1^{++}(x^+, \rho)(\partial_+\rho) \ +$$
$$B_2^{++}(x^+, \rho)(\partial_+\rho)^2 + B_3^{++}(x^+, \rho)(\partial_+^2\rho) = 0, \tag{45}$$

where B_i^j's are expressions of the fields $f_{st}(\rho)$, $A_{st}(\rho)$, $H(x^+, \rho)$, $A(x^+, \rho)$ and their derivatives very complicated to be presented here. Following a procedure similar to the one adopted for the static case we eliminate the derivatives of the field ρ using the ansatz in (42) and the form of the third of the equations in (45).

The resultant system is a highly non-linear partial differential system hard to be solved. Instead having in mind the method which has been exposed in the case of the static problem we seek solutions in κ-series expansion. The general solution of this system for the fields up to order κ^2 comes to be the following

$$\phi(x^+, \rho) = \rho(x^+, x^-) + \frac{\kappa c_0}{4F_0^2(x^+)}, \tag{46}$$

and

$$e^{-2\rho} = c_8 e^{-\kappa c_0/2F_0^2(x^+)} + F_0^2(x^+)(c_7 - x^-)e^{-\kappa c_0/2F_0^2(x^+)} + (8c_4\kappa + 4\kappa^2 c_6)F_0^2(x^+), \tag{47}$$

where c_i's are integration constants. In the above the integration constant c_8 has to be non-negative, since when $\kappa = 0$ coincides with the mass parameter of the Schwarzschild solution. Furthermore the function $F_0(x^+)$ satisfies the equation

$$F_0'(x^+) = \frac{1}{2F_0(x^+)}e^{\kappa c_0/2F_0^2(x^+)} \tag{48}$$

with solution given in implicit form by

$$x^+ = F_0^2(x^+)e^{-\kappa c_0/2F_0^2(x^+)} - \frac{\kappa c_0}{2}E_i\left(-\frac{\kappa c_0}{2F_0(x^+)^2}\right), \tag{49}$$

where E_i is the exponential integral function. from the desired asymptotic behaviour of F_0, $F_0(x^+) \to 0$ as $x^+ \to 0$ and $F_0(x^+) \to \sqrt{x^+}$ as $x^+ \to +\infty$, the constant c_0 has to be positive and thus F_0 is a monotonic function of x^+.

For the Ricci scalar $R = 8e^{-2\rho}\partial_+\partial_-\rho$ we get the expression

$$R = 4e^{2\rho(x^+, x^-)}\{c_8 e^{\kappa c_0/2F_0^2(x^+)} - 2c_0(2c_4\kappa^2 + c_6\kappa^3)\} \tag{50}$$

At this point we notice the close similarity of the above solution with the exact solution found in [5] where the role of the classical tachyon configuration is played here by the function $[F_0(x^+)]^{-1}$. This solution describes a black hole geometry. The event horizon is at some $x^- = constant$. As far as the physical singularity is concerned we see that besides the singularity coming from the zero of $e^{-2\rho}$ in (47) we have the curvature blowing up also at $x^+ = 0$. As it is explained in [5] this is not a naked singularity but rather an initial one. This extra singularity is a general feature of the solution and is inherently connected to the deviation from staticity. In (46) the singularity appears in κ-order since we introduce time-dependence to this order. This could be implemented at higher orders in the κ-expansion and in this case we can see that the equation (49) for the corresponding F_0 remains the same. If for example we change the relation (46) keeping the static contribution up to κ^2 order then one can see that exactly the same time dependent term $\frac{1}{F_0^2}$, with F_0 given by (49), emerges at κ^3 order, while all the static contribution to the dilaton field disappears. In fact this is the only kind of time dependence permitted if we insist on the analyticity in the parameter κ. Our analysis cannot exclude solutions non-analytic in κ but such solutions cannot describe an evolution scheme between two static black hole geometries since these, are analytic in κ.

5 Conclusions

In this work we consider solutions of two-dimensional dilaton gravity with R^2 term which can be viewed as backreaction term from quantization of matter. Of course this term has interest by itself considered as R^2 gravity but we emphasize its quantum origin and the fact that, in the semiclassical approximation, it does not contribute to the Hawking radiation [2]. We confirm the static solutions found in [3] working in the conformal gauge and using light-cone coordinates. As a bonus of the new analysis we find that the black hole covers a part of the physical space of the Schwarzschild geometry. In particular the horizon (and eventually the sigularity also) lies in this region. Futhermore we find that the expansion in powers of the coefficient of the R^2 term describes well the qualitative features of the new geometry. We address also the time dependent problem. We seek for solutions analytic in κ since such solutions have the possibility to describe an ordinary evolution scheme between the Schwarzschild black hole and the backreacted static one. Nevertheless we find that allowing time dependence, even in the manner described above, we have a drastic change of the geometry. A new singularity appears at $x^+ = 0$. We note that the solution found has black hole structure and this reflects the fact that the term included in the action does not contribute to the thermal radiation. On the other hand the calculation of the Bogoliubov coefficient in this background [10], shows that we have a non-thermal spectrum and hence a thermodynamic instability of the system. We remark here that the horizon of the static R^2 black hole, being removed in the physical spacetime region of the Schwarzschild black hole, can in principle cover this extra singularity. This in connection with the evolution caused from the thermodynamic instability shows that the backreaction effects are very important for the black hole evolution. We conjecture that these effects may prevent the full evaporation of the black hole [11]. Nevertheless the study of such a quantum evolution or any other possibility like the one described in [10] demands a more general context beyond the semiclassical approximation adopted here.

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